"A Dynamical Theory of the Electric and Luminiferous Medium. Part III. Relations with Material Media." By JOSEPH LARMOR, F.R.S., Fellow of St. John's College, Cambridge.

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(Abstract.)

1. It was shown by Maxwell that the theory of electric current systems flowing in rigid conductors could be formulated dynamically if the current in each circuit, which is supposed constant all round it, is taken as a generalised velocity: a formal development in his manner of the properties of the current system, from the single dynamical foundation of least action, was given in the first of the present series of papers. It is implied in such a view that the positions of the linear conductors and the intensities of the currents that are flowing in them control completely, it may be in an entirely unknown manner, the motions that are going on in the surrounding æther; just as in the cyclic irrotational motion of a perfect fluid the positions of the rigid cores round which the circulations take place, and the amounts of these circulations, determine the motion. But it was pointed out that, whereas the vorticity of such a core is dynamically a momentum, on the other hand in order to satisfy the facts the currents in the circuits must be considered dynamically as velocities; so that there is not any real analogy between the two cases.

This simple dynamical formulation is no longer available when attention is not confined to complete rigid circuits, for example when the mechanical forces acting on a portion of a circuit are in question. It is insufficient also when the electromotive force (integrated electric force) between two points of a circuit carrying a current is to be discussed, that quantity being realisable and measurable by bridging the two points through the incomplete conducting circuit of an electrometer.

A knowledge of the electric force at a point, as well as of its integrated value round a circuit, is also essential when the current in the circuit is in part conducted and in part arises from changing electric displacement: it is thus essential in the theory of electric waves and radiation, though unimportant in ordinary electrodynamic applications; and for this reason that part of electric theory remained unsettled until Hertz showed how to produce and investigate free electric waves.

Previous to that time, the theory had been completed hypothetically in various ways, by extending the range of the dynamical formulæ (with such generalisations as were admissible) from the complete

rigid circuit and the complete circuital current to the geometrical elements of which they are composed. The way was prepared for this by F. Neumann, by his discovery of the electrodynamic potential formula, from which he deduced by a uniform analytical process the mechanical and electric forces in complete rigid circuits, as previously formulated by Ampère and Faraday. Then Helmholtz took up and extended Neumann's theory, introducing Maxwell's principle that the formula of Neumann represents the kinetic energy of the system of current elements, to which Lagrange's general dynamical equations, or more precisely the action method, can be applied. His main aim was to include in this potential theory the conception of dielectric currents introduced by Faraday: but he was at the same time led to generalise Neumann's formula so as to include the possibility of waves of electric compression as well as transverse electric waves in dielectric media: the existence of the former type of waves Hertz's discoveries have disproved.

This theory of von Helmholtz thus originated by way of elucidating, from the older standpoint, the æther scheme advanced by Maxwell in the shape of a system of analytical relations, which were the mathematical formulation of Faraday's views, and involved also a unification of the electrical and optical functions of the æther. The equations of Maxwell include implicitly a definite theory of all electromotive phenomena in open circuits *at rest*, a fact which is not very obvious for the form into which they were thrown in Maxwell's "Treatise," but which is plainer for the form of two conjugate circuital relations into which he had previously cast them as being the simplest and most direct formal expression of the theory, but which he left aside in favour of a conception of electrodynamic momentum or vector potential that was intended to connect the equations directly with dynamical principles: this simple formal specification of ethereal relations has since been restated and utilised by Heaviside and Hertz. These Maxwellian fundamental equations, being purely electromotive, gave no direct explanation of ponderomotive forces: for that purpose the theory had of necessity to take on a dynamical form. A formula for the distribution of the electric energy throughout the æther was suggested by various considerations, and from it an attempt was made by the methods of general dynamics to establish the laws of the force exerted by the æther on the different parts of conductors conveying currents: and it was natural that the same procedure should be extended to an attempt to place the fundamental formal equations of the æther itself on a dynamical basis. But what was lacking for the satisfactory accomplishment of this purpose was a definite and consistent idea of how the electric charges and currents in the matter established a hold on the æther. In the absence of this, Maxwell had to rely, when actions on portions of

circuits were under consideration, on the notion of a current element, and to derive the formulæ from that conception with such degree of definiteness as was possible. It was this imperfect dynamical method which it was the aim of Helmholtz to discuss and elucidate in the analysis above mentioned.

In so far as these theories were dynamical they all involved current elements: but the criticism of the second of the present memoirs is held to show that a current element is not a legitimate dynamical entity, in that the forces strictly derivable from that assumption, both mechanical and electric, are in disagreement with experimental knowledge: and the reason is indicated, namely that the method of current elements forms an incomplete specification of the phenomena, inasmuch as it gives no account of how a current is induced within an element of volume of the matter by separation of the two electricities under the action of the electric force. This led to the introduction of the mobile electron or atomic charge of electricity as the true physical element, and to a dynamical theory of molecular type which is held to be self-consistent and in full agreement with experimental knowledge, and which may be regarded as in a manner a final development of the Weberian notion of moving electric particles. The *true* electric current of moving electrons is thus made up of a current of conduction and a current of dielectric polarisation: it does not flow in closed circuits, but if there is added on to it a quantity called the æthereal displacement current, which is not a flow of electricity at all but a flux of elastic displacement of the æther, there is obtained the *total* circuital current of Faraday and Maxwell. This more precise three-fold specification of the total current, in place of the two-fold specification of Maxwell which ignores the physical distinction between the polarisation current in the dielectric matter and the displacement current in the æther occupying the same space, introduces notable differences as regards ponderomotive forces, moving material media, electric radiation, and in other respects. In the main the results correspond with Faraday's geometrical mode of specification by means of tubes of induction; they agree more closely with the scheme derived by Maxwell from direct consideration of his mechanical model of the electric field than with the later analytical theory of his "Treatise."

The general equations of the electric field, when it contains moving material bodies, dielectric or conducting, are formulated on these principles. They are applied to the problem of the uniform rotation of a charged conductor, or a dielectric body, in a magnetic field: to the influence of motion of a material medium on the velocity of radiation that is passing across it: and to the influence of steady translation of a material system through the æther on its configuration and the distribution of its electric charges.

2. The object of the present memoir is to further develope and apply this conception of the relations of æther and matter, which has thus been shown to form a working basis for optics and electro-dynamics. According to this scheme the æther is the seat of the elastic transmission of electric and magnetic force: but the pondero-motive forces acting on its electrons or strain-centres, which represent matter, are derived directly from the energy without any reference to such transmission; and this has been taken as an objection to the theory. We are therefore led to a critique of the sufficiency of the principle of step-by-step transmission or contact action as a basis for a complete connected representation of the play of physical agencies.

With a view to thus attaining more precise ideas as to what constitutes ultimate physical explanation, the properties of a material elastic solid medium involving in its constitution centres from which intrinsic strain spreads out into the surrounding parts, are noticed. The theory of fluid vortices, irrespective of any claim to form a natural representation of material phenomena, has been of fundamental service as an illustration of what kind of interconnexion it is conceivable to assume between matter and the universal æther: in vortex-atom theories the fundamental reality as regards pressure and inertia is transferred to the continuous fluid æther, and the properties of the atomic matter, which had been the original source of dynamical suggestion, reappear as secondary or derivative. Now just as a vortex atom is a state of intrinsic motion which can travel or flit through the fluid, independently of, and in addition to, motion of the fluid itself, so also can a strain-centre in an elastic solid medium move about independently of the medium.* Its motion forms an element beyond and in addition to the changing strain impressed on the parts of the solid medium by external influence. It would be easy to construct a solid body involving such strain-centres, and this circumstance gives an impression of actuality which may be felt as wanting in an abstract theory: the material of the solid would illustrate æther, the strain-form would represent an essential of matter. Such a representation is a step beyond and outside the ordinary idea of an elastic medium as the mere vehicle for transmission of forces: that notion, effective in its own sphere, would not give any account of where or how these forces originate, or what is the nature of the connexion by which the matter gets a grip upon the impalpable æther. Nor was it to be expected that the mere idea of contact action, which has come to be interpreted as an idea of elastic transmission, could of itself give account of the actions of

* The analogy is not quite complete; for the vortices move so that their cores are always made up of the same portions of the medium, while movement of a strain-centre does not to any extent carry the medium with it.

which it is only the mechanism of transmission. It is here maintained that the whole of the action between different portions of matter cannot possibly be represented as transmitted by the æther in that manner. It would even be a fair defence of this position to claim that the *onus* of demonstration lies on those who assert the opposite: but the material elastic medium, above mentioned, pervaded by intrinsic strain-centres, furnishes a crucial illustration. The parts of that medium itself are in motion on account of the changing strain; while in addition the intrinsic strain-centres put each other into motion across the medium (as vortices do across an intervening fluid) by their mutual actions. The theory of contact action so called, or elastic transmission, is not wide enough to include all this: the dynamical interactions between the strain-centres are no more transmitted by elastic action than the interactions between fluid vortices without solid cores are transmitted by fluid pressure: though in each case that agency is a necessary concomitant to the dynamical effect. The really absolute thing in dynamical explanation, that on which this principle of elastic transmission or contact action has itself been built up by Lagrange and Green, and without which it could not have assumed a precise mathematical form, is the scheme of fundamental dynamical notions connecting inertia and force, in their modern generalised aspect which formulates them under the energy principle. The main problem of transcendental physics is to assign the nature of the ultimate medium or scheme of relations which combines physical phenomena into a unity, in whose relations these dynamical notions have their scope: and it is only the prejudice of education that would keep, in this wider field, too close to the ideal of mechanical transmission in a homogeneous elastic solid.

In so far as we can collect the various dynamical principles into a simple formula, we shall attain the security that all the trains of results that come from that formula are consistent among themselves, and form parts of a single interlaced scheme. Such a formula was dimly foreshadowed by Maupertuis as the Principle of Least Action, and after various elucidations by Euler was finally established in its generality by Lagrange for ordinary material systems. The ultimate unification of physical theory, which transcends and includes ideas of contact action and other partial explanations, would thus lie in the formulation of the energy function of the æther, including matter, in a manner suitable for the application of the action analysis to the correlation of the observed phenomena. To assume that this will ever be accomplished absolutely and completely is nearly the same as to assume infinite capacity in the human understanding: but exact knowledge of inanimate nature comes by a process of analysing and classifying into types the main physical agencies as they present themselves in bulk to our senses, and it is a

legitimate and feasible problem to seek out these aspects of the underlying unity which are the cause of the interactions and correlations of these agencies, so far as they have yet been unravelled. The mere recognition of the precise order that reigns in the larger workings of the unlimited diversity that constitutes matter is itself the strongest argument that the common basis of the varieties of matter involves something simple and universal in its relations, with which we are really in physics more intimately concerned than with the infinity of arrangements and collocations that the molecules of matter can individually and collectively assume.

It follows from the analysis of the second of these memoirs that the action formula can be completely expressed and developed in a molecular theory sufficiently refined to take account of each electron separately; also that the main outline of the ordinary electrodynamic theory, for finite systems of bodies treated as continuous aggregates, can be developed from the action formula transformed so as to be expressed in terms of matter in bulk, when the currents are specified as circuital and the different regions are homogeneous as regards electric and magnetic polarisation. But in questions of details of mechanical action on matter, especially when in motion, and in questions involving the distinction between the true current carried by the matter and the total circuital current, and also involving heterogeneity in the dielectric, the pure dynamics does not suffice, and recourse must be had to direct processes of averaging, such as are necessary in other domains of molecular theory, as for example the theory of gases.

3. The utility of an elastic solid model of the kind above described is not to represent the æther, but to enlarge our ideas: for the optical phenomena show that the elasticity of the actual æther is of rotational, not distortional or elastic solid, type. It is however also explained in the memoir how, out of matter gyrostatically dominated, it is theoretically possible to construct a model which will represent the æther itself and its electrons for any assignable time, though not for ever: the properties of the model there described would gradually fade away, just as if matter were not eternal. A main difficulty in designing such a representation lies in the circumstance that it must possess the property of perfect fluidity for irrotational motions.

The distinction between the features of elastic transmission in the æther and in material elastic substances is brought out. The former medium is a pure *continuum* of which elasticity, inertia, and continuity of motion, are the sole ultimate and fundamental properties. Matter on the other hand is made up of discrete atoms or singular points in the æther; it has the inertia which belongs to these singularities individually; it has elasticity on account of their

interactions through the æther; its continuity of motion is, in the case of fluids, of limited character, being maintained only by viscosity and other such causes. The elasticity of matter in bulk is to be based on the distribution of organised material energy per unit volume: the material energy depends on the relative positions of the atoms, therefore this organised or mechanical part of it depends on the change of their mutual configurations expressed with reference to the deformation of the element of volume, that is on the strain. It follows, as Green was the first to show with logical simplicity and precision, that the stress in an element of volume is self-conjugate. But none of these ideas have any application to the æthereal *continuum*: in it there is no question of change of mutual configuration of physical parts; the energy of strain is not thus restricted to be a function of deformation only, in fact after MacCullagh it assumes the geometrically more simple form of a function of the absolute position, more precisely of the rotational displacement, of the element; the resulting stress need not be self-conjugate,—nor is it in fact so even in a material medium that has gyrostatic quality. An electric field would consist of rotational strain in the æther, a magnetic field of irrotational flow, each in actual cases extremely slight: the motion involved in a permanent magnetic field combined with a permanent electric field would not become jammed in course of time, because it can relieve itself by a slight separation of free electrifications, which will again neutralise each other after this object has been attained.

4. An atom of matter has been represented by a collocation of electrons describing stable orbits round each other. The discussion of the internal vibrations of such an atom and the consequent radiation will follow the lines of Laplace's general analysis of the oscillations about steady motion of a system of connected bodies like the Solar system. When the gyrations or orbital motions are sufficiently rapid, there will be two types of vibrations produced by disturbance of the system; very rapid ones which radiate light, and very slow ones like the precession of a spinning top which do not involve appreciable internal deformation of the system. In gases it is only these latter that would be excited sensibly by the comparatively gentle encounters between the molecules: these are in relation to the thermal energy, but are only in indirect connexion with radiation. The difficult outstanding problem of the theory of gases, that namely of the connexion between temperature and internal thermal energy, involving the relation of the two specific heats, would on this view take on a form different from the usual one. In various other respects, a recognition that the motions which constitute heat are not the vibrations which feed radiation seems to extend and improve the capabilities of molecular theory.

Another field in which the influence of a gyratory character in the molecule might be expected to be prominent is that of optics, more particularly the influence of matter as it appears in refraction and reflection. As an introduction to this subject, Lorentz's law of the relation between refraction and density is worked out; the argument is purely statical and independent of the constitution of the molecule, and closely follows a cognate investigation of Clausius as I afterwards discovered; it is however retained, as it appears to be exempt, within its proper scope, from the objections which are valid against other modes of demonstration of that law that have been proposed. When we pass on to discuss dispersion, the forced vibrations of the molecules come in: and these will be of different type according as the molecule is taken to be a system vibrating about a position of rest as has hitherto been tacitly done in optical theory, or a system vibrating about a state of steady motion as it is here required to be. The main result, for a medium devoid of non-selective opacity such as would arise from conduction, is that the Lorentz refraction equivalent $((\mu^2-1)/(\mu^2+2)\rho$ (not μ^2-1 as in the usual dispersion theories) is an additive physical constant, equal for each simple medium to $\Sigma g_r/(p_r^2-p^2)$, where p_r is a natural vibration-frequency for the molecule and g_r is a related constant. The only simplification that comes in when the gyratory quality is absent is that then g_r is necessarily positive: in the present case the reasons for taking it to be positive are not so conclusive. If in any term g_r were negative the character of the anomalous dispersion near the corresponding absorption band would be the opposite to that indicated by Kundt's law, which has hitherto always been observed to hold good.

The only way that is *à priori* unexceptionable for determining the complex index of refraction of a strongly absorbing medium such as a metal is Kundt's method of deviation by thin prisms: this gives only the real part of the index, but it is shown that by taking advantage of oblique as well as of normal incidence approximate values might be obtained for the other part as well. The fair agreement of Kundt's values with those derived from experiments on polarisation by reflection is however a confirmation that surface films are not seriously operative in the latter method, which has yielded both parts of the index. If there were no non-selective absorption, the curve representing the real part of the index would rise to infinity near each absorption band, then fall straight down to the axis, coincide with the axis for an interval, and finally again rise above it. But when there is also general absorption the curve will turn back before reaching infinity, and it will not descend as far as the axis, while the sharp corners will be eased off. These characteristics are precisely those of anomalous dispersion, for example of

Pflüger's recent dispersion curves for solid fuchsin and other selectively absorbent solid substances; but there is absolutely nothing in these general features that would not fit one theory of dispersion as well as another,—they all arise from the mere notion of sympathetic vibration. The complete values of the index along the spectrum, not merely those of its real part, must be available before any preference can in this way be established.

5. A main feature of the interaction between æther and matter consists in the bodily mechanical forcives exerted on electrically and magnetically polarised material media. In the theory a transition has here to be made between the mere aggregate or sum of the varying energies of the individual molecules of a medium, and the co-ordinated and averaged part of this sum which is the energy pertaining to the element of volume of the medium in bulk: this further involves the definite enunciation of a general principle in molecular mechanics which has hitherto found an application, and that a restricted one, only in the theory of capillary attractions. In a polarised medium, a distinction has thus to be drawn between the averaged individual energies from which the forces polarising the separate molecules are derived, and the organised mechanical energy of the medium as a whole, which is the aggregate of these energies after the local parts arising from the neighbouring molecules have been excluded. In this mechanical energy the bodily forces on the medium in bulk are involved: it must therefore be expressible as an analytical function of the configuration of the medium in bulk, for otherwise perpetual motions could supervene. In the study of the mechanical actions in material media, consideration of the properties of the molecules is available as a guide towards the mathematical form of the function which represents the distribution of the mechanical energy of the forces acting on the element of mass: but the province of molecular theory is ended in this general survey, and the actual values of the coefficients in the energy function must be determined by observation and experiment.

In an electrically polarised material medium an expression for the distribution of this mechanical energy is obtained, and the bodily applied forces in the material are derived from it, the tractions exerted on an interface between two media being deduced from the forces that would act on a layer of gradual transition which in the limit is taken indefinitely thin. The result is that in a medium whose molecules are polarised to intensity i' by a field of electric force F, and isotropic so that i' and F are in the same direction, there is a bodily force $(d/dx, d/dy, d/dz)\int i'd\mathrm{F}$, which could be balanced by a hydrostatic pressure $-\int i'd\mathrm{F}$, and there is also a normal traction on each interface equal to the difference in the values of $-2\pi n'^2$ towards each side, where n' is the component of i' along the normal. Thus

when a fluid medium is in equilibrium there must exist in it a hydrostatic pressure $\int i' d\mathrm{F}$, and in addition on each interface a traction $2\pi n'^2 + \int i' d\mathrm{F}$ along the normal towards each side, arising from other than electric causes and balancing the electric forcive: so that to maintain mechanical equilibrium, an extraneous normal traction $2\pi n'^2$ towards each side of each interface is alone required.

6. This result differs from that of von Helmholtz's investigation, also based on the method of energy: the origin of the discrepancy is traced to the circumstance that a single continuous energy-function cannot serve for the complex medium æther *plus* matter. This difference goes to the root of things, especially in optical theory, even in cases where the resulting expressions present no difference in form. Variation of the physical constants of the medium arising from the strain involved in the virtual displacement is also included by von Helmholtz in the deduction of the mechanical forcive, thus introducing effects which are here held to be more consistently explained as physical changes arising from the molecular action of the polarisation.

Of the purely local part of the total energy of a molecular medium, there is a regular or organised portion depending on the deformation of the material in bulk, which is the energy of the mechanical stress that compensates the applied mechanical forces: the remaining, usually wholly irregular, part finds its compensation in other interactions between neighbouring molecules, which may reveal themselves in the aggregate in alterations of the local physical constants of the material as well as of its volume and other dimensions.

But in the circumstances of a medium electrically polarised this *residuum* itself involves a part which is regular in each element of volume, arising from the regularity in the orientation of the molecules which act on each other in that element. The mutual forcive thence originating may be expressed, though there is not much object in doing so, as regards the interior of an isotropic medium, as an internal molecular stress related to the lines of polarisation. When the distance between the effective poles of a molecule is small compared with that between neighbouring molecules this stress is a tension $\frac{4}{5}\pi i'^2$ along the lines of polarisation together with a pressure $\frac{2}{3}.\frac{4}{5}\pi i'^2$ uniform in all directions at right angles to them: it is to be considered as balanced locally by cohesive reaction.

Under all circumstances, the forces between neighbouring molecules produce and are compensated by change of the relative configuration of these molecules; they thus produce change of the *local* physical constants of the material, and also *local* intrinsic change of volume and other dimensions, all which are proportional to the square of the polarisation; but they contribute nothing directly to the mechanical stress transmitted by the material in bulk. In a solid material, however, these intrinsic changes of configuration of

the elements of volume may not fit together consistently with the continuity of the substance, and thus secondary strains may be produced which will complicate the problem. But in fluids, in which alone experiment is feasible, no such complication can occur.

The position is aptly illustrated by the ideally simple case of a perfect gas polarised in an electric field. The mechanical forcive due to the polarisation of the gas as a whole is there compensated by change of pressure, which is transmitted. There is another regular part of the forcive which arises from actions between neighbouring molecules, so that those in the line of polarisation attract and those in lines at right angles to it repel each other, after the manner of little magnets; and this, which differs from the former by being proportional to the square of the polarisation instead of its first power, is not experienced as a mechanical force, because it is wholly compensated on the spot where it originates by slight change in the ordinarily fortuitous distribution of velocities of the molecules of the gas, by which its constitution acquires an axial character with reference to the line of polarisation so that the pressure is no longer quite the same in all directions.

7. The relation is explained which exists between this *organised* or *mechanical* energy and the *available* or *free* energy of thermodynamics. The principle of available energy, which itself is a direct consequence of the negation of perpetual motions, or rather of the negation of the unlimited availability of diffuse thermal energy, is the single essential foundation of that science. It is pointed out, that if we had no direct perception of temperature through our senses, this negation of perpetual motions would necessitate the introduction of that quantity into physics, somewhat in the same way as potential is introduced into electrical theory, and would yield a demonstration of its fundamental property. Instead then of making attempts, by the aid of special molecular hypotheses of more or less problematical character, to obtain a purely dynamical definition of temperature and an analytical demonstration of Carnot's principle, it is suggested that it is more philosophical to recognise that no physical scheme of matter and molecular action is conceivable that would involve perpetual motions (in the above sense) of matter in bulk, and to base the pure theory of thermodynamics and thermochemistry directly on this postulate.

The case of homogeneous fluid media, which acquire energy of polarisation of any kind when in a field of force, is considered in a general manner: it is shown that such media will be in mechanical equilibrium provided an extraneous traction along the normal is applied over each interface between them, of intensity equal to the difference of the densities of the mechanical energy of polarisation on the two sides of the interface.

Osmotic pressure is related to the total available energy, not merely to the mechanical part of it: the limiting or maximum value which it cannot exceed is equal to the change in available energy produced per unit volume of transpiration across the partition. An ultimate deduction of van't Hoff's law of analogy between osmotic and gaseous pressure is offered, on the foundation of the principle of available energy, which is independent of any assumption as to the character of that pressure whether purely kinetic or otherwise: if this be accepted, it will follow that no inference as to the physical state of the dissolved substance, except as regards its degree of effective dissociation, is deducible from the osmotic law. It appears to have escaped general notice that what was virtually a prediction of the law for the cases of dissolved gases is involved in the equations of von Helmholtz's discussion of the influence of dissolved gas on electromotive force, in which however the argument is based on Henry's law of solution; the law itself had indeed been formulated explicitly on similar theoretical grounds by Willard Gibbs still earlier. The influence of an electric field on osmotic pressure between dielectric fluids is estimated: this involves by cyclic processes the influence of an electric field on the vapour pressure and on the freezing point of a dielectric liquid. Some considerations connected with the nature of the process of ionisation are brought forward. The laws of chemical equilibrium, as developed by Guldberg and Waage and by van't Hoff, are placed in relation to the principle of available energy. That method is also applied in a discussion of the electromotive force of a voltaic cell, and especially of the dissipative part which is established by steady finite diffusion between solutions of different concentrations.

8. A thermodynamic application which possesses interest, both from the light it throws on the nature of magnetism and from the circumstance that in it the heat supply is calculated indirectly from the magnetic energy that runs down, is the relation between magnetic susceptibility and temperature in substances not in the very susceptible or ferromagnetic condition. According to the Weberian theory, which fits in with the present view, diamagnetic energy which is not compensated mechanically goes to the induction of Amperean currents in the molecules; while paramagnetic energy not thus compensated goes to orientating the molecules, and thus into heat. It follows that the diamagnetic coefficient is independent of temperature: on the other hand it is shown that the paramagnetic coefficient should vary inversely as the absolute temperature. These laws were discovered experimentally by Curie, who finds from a very extensive investigation that they have the same order of accuracy at sufficiently high temperatures as the ordinary gaseous laws: at lower temperatures and in ferromagnetic substances the

control of the polarised molecules arises in appreciable part from the magnetic interaction of their neighbours, thus vitiating the law as well as introducing effects of hysteresis. The well-known model of Ewing would thus represent an ideal perfect ferromagnetic in which the control arises wholly from the latter cause.

In application of the previous results as to how far physical actions can be considered as transmitted across the æther by elastic stress, the conditions are formulated under which the correlative principle utilised by Poynting is valid, that the actual rate of change with time of the organised or mechanical energy within any region is expressible explicitly as a surface-integral over its boundary.

The mechanical effects of light-waves are reconsidered in the light of this molecular theory. The conclusion is reached that such effects are wholly associated with absorption of the radiation, that no influence of perfectly transparent media on radiation can provoke a mechanical reaction. There is a mechanical force acting on an absorbing mass, in the direction of the incident radiation and equal to $E(1-m^{-2})$, where E is the energy absorbed per unit time and m is the real part of the index of refraction. Partial analogies are furnished by the mechanical effects of Hertzian radiation on a medium built up of conducting linear circuits, and of sound waves on a medium formed of a system of resonators.

As an application of the law of the mechanical forcive on dielectrics, the changes of dimensions of a condenser under electrification are considered. The problem is found to admit of exact solution if the condenser layer consists of a closed sheet, of any form, but of uniform thickness. In that case the mechanical stress in the material of the sheet proves to be simply of the type of the Faraday-Maxwell stress. The theory is compared with Quincke's experimental results: their main features are verified, including those which led Quincke to assign a wholly non-mechanical origin to the effect: but something less than half the change of volume remains over as an intrinsic electric deformation, not due to the transmitted mechanical forces.

Finally a series of practical illustrations of the mechanical theory are treated, some of which have already been employed for experimental measurement, and which are capable of still further application. The mechanical circumstances attending the refraction of uniform fields of electric force by fluid media are developed. The theory of various arrangements for measuring electric tractions and pressures in fluid dielectrics is worked out. The effect of an electric field on the velocity of ripples on the surface of a conducting or a dielectric fluid is determined: as also are the relations of electric polarisation to vapour tension and fluid equilibrium. The internal mechanical forces in a complete magnetic circuit are examined, and

also the traction between the interfaces when it is divided: and the mode of calculation of the stress in a sphere of iron in a uniform magnetic field is indicated, agreeing for this case with Kirchhoff. The mutual influence of stress and magnetisation is analysed, with reference to the experimental investigations of Bidwell.

Throughout the memoir care is taken to dispense, as far as possible, with detailed algebraic processes, which are essential for special computations and verifications, but are best evaded in the discussion of general principles. Most of the discussion is also independent of the rotational æther scheme: the great advantage of an interlacing hypothesis of that kind, which remains even when it is only provisional, is that it gives an insight into the character of the formal relations that are possible or probable between the actual physical quantities involved in it.

"On a new Method of Determining the Vapour Pressures of Solutions." By E. B. H. WADE, B.A., Scholar and Coutts-Trotter Student of Trinity College, Cambridge. Communicated by Professor J. J. THOMSON, F.R.S. Received April 26,—Read May 13, 1897.

(Abstract.)

The measurement of the vapour tension of solutions has hitherto been attended with serious errors. This is especially true of the statical methods, which have alone been capable hitherto of furnishing results at temperatures between 60° C. and 100° C.

The apparatus employed in this research was in conception similar to that described by Sakurai,* except that it was in duplicate, a divided steam supply passing through two U-tubes placed in parallel. It differed, however, from his apparatus in several important features, which cannot adequately be described in a paper of these dimensions. Suffice it to say that the pressure on the contents of the two U-tubes, being the *same*, could be adjusted to any convenient value, and that the method of thermometry being differential,† the difference only of the boiling points of pure water and solution, in their respective U-tubes was recorded. Two series of experiments were made at a pressure of 760 mm., in one of which a small external heat supply was used to compensate the condensation in the U-tubes, and a second in which this was dispensed with, and constancy in the amount of liquid operated on (which proved to be of the first importance) was

* 'Chem. Soc. Journ.,' 1892.

† Identical with that employed for many years by Mr. Griffiths.